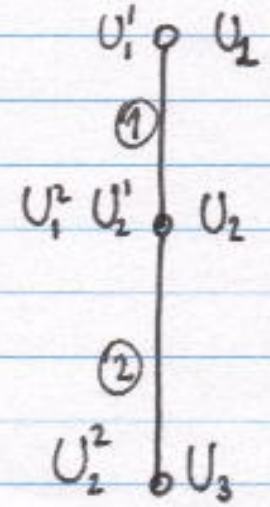
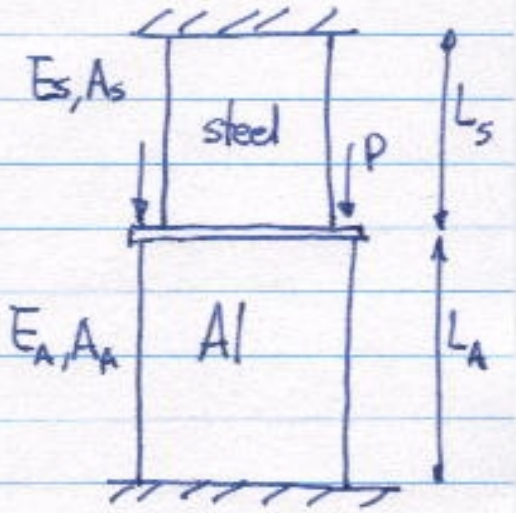
Solution set

③ 9.7

Model: 2 linear finite elements:





Stiffness matrices (element): from problem ②

$$K^1 = \frac{E_S A_S}{L_S}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad K^2 = \frac{E_A A_A}{L_A}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global stiffness

$$K = \begin{bmatrix} \frac{E_S A_S}{L_S} & -\frac{E_S A_S}{L_S} & 0 \\ -\frac{E_S A_S}{L_S} & \frac{E_S A_S}{L_S} + \frac{E_A A_A}{L_A} & -\frac{E_A A_A}{L_A} \\ 0 & -\frac{E_A A_A}{L_A} & \frac{E_A A_A}{L_A} \end{bmatrix}$$

Force vector: $R^1 = \begin{Bmatrix} 0 \\ 2P/2 \end{Bmatrix}$, $R^2 = \begin{Bmatrix} 0 \\ 2P/2 \end{Bmatrix}$

→ Global, $\boxed{R = \begin{Bmatrix} 0 \\ 2P/2 \\ 0 \end{Bmatrix}}$

FE equations $\begin{bmatrix} K \end{bmatrix} \begin{Bmatrix} U^1 \\ U^2 \\ U^3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 2P \\ 0 \end{Bmatrix}$

B.C. : $U^1 = U^3 = 0$, from 2nd row

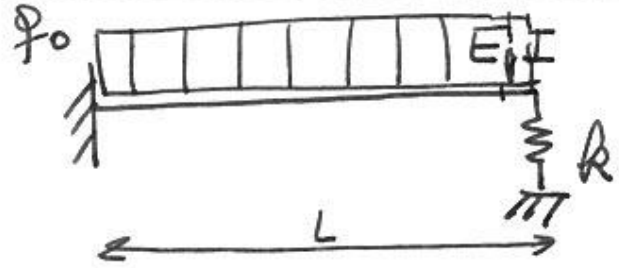
$$-\frac{E_S A_S}{L_S} 0 + \left(\frac{E_S A_S}{L_S} + \frac{E_A A_A}{L_A}\right) U^2 + \left(-\frac{E_A A_A}{L_A}\right) 0 = 2P$$

$$\Rightarrow \boxed{U^2 = \frac{2P}{\frac{E_S A_S}{L_S} + \frac{E_A A_A}{L_A}} = \frac{2P L_S L_A}{L_A E_S A_S + L_S E_A A_A}}$$

Replacing: $P = 50\ kN$, $E_S = 200\ GPa$, $A_S = 60\ mm^2$, $L_S = 10cm$
$E_A = 70\ GPa$, $A_A = 600\ mm^2$, $L_A = 20cm$

→ $\boxed{U^2 = 0.3\ mm}$

HA08 - P4 (9.13 from textbook)



Use one element

$$\frac{2EI}{L^3}\begin{bmatrix} 6 & -3L & -6 & -3L \\ & \text{etc.} & & \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ U_3 \\ U_4 \end{Bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 0 \\ 0 \\ U_3 \\ U_4 \end{Bmatrix} = \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \\ 0 \end{Bmatrix}$$

K^e (beam) — spring stiffness

extract equations:

$$\begin{cases} K_{33}\,U_3 + K_{34}\,U_4 + k\,U_3 = R_3 \\ K_{43}\,U_3 + K_{44}\,U_4 = 0 \end{cases}$$

$$K_{33} = \frac{12\,EI}{L^3} \qquad K_{34} = K_{43} = \frac{6\,EI}{L^2} \qquad K_{44} = \frac{4\,EI}{L}$$

$$R_3 = \frac{q_0 L}{2}$$

Solve system: $\left(K_{33} + k\right) U_3 - \frac{K_{34} K_{43}}{K_{44}} U_3 = R_3$

$$U_3 = \frac{R_3}{K_{33} + k - \frac{K_{34}^2}{K_{44}}} = \frac{\frac{q_0 L}{2}}{\frac{12EI}{L^3} + k - \frac{\left(\frac{6EI}{L^2}\right)^2 L}{4EI}}$$

$$= \frac{q_0 L^4}{2EI\left(12 - 9 + \frac{kL^3}{EI}\right)} = \underline{\frac{q_0 L^4}{6EI\left(1 + \frac{kL^3}{3EI}\right)}}$$

$$U_4 = -\frac{K_{43}}{K_{44}} U_3 = -\frac{6\cancel{EI}}{L^3} \frac{k}{4\cancel{EI}} U_3 = -\frac{3}{2L} U_3$$

$$= \underline{-\frac{q_0 L^3}{4EI\left(1 + \frac{kL^3}{EI}\right)}}$$