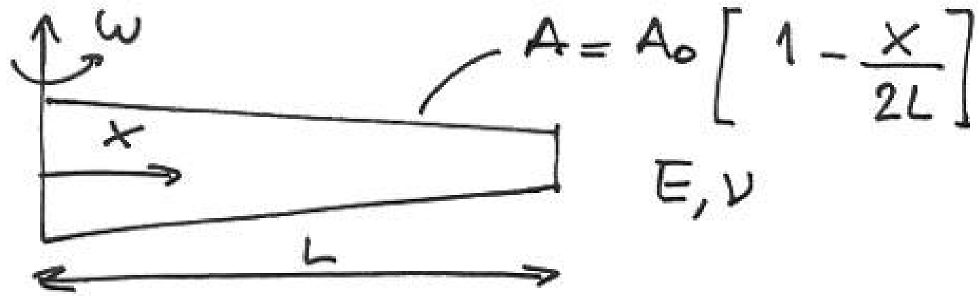
16.21 Final 2002 - Practice Session



Determination of the load on blade

$$a(x) = \omega^2 x$$

$$f(x) = \rho\, A(x)\, a(x) = \rho \omega^2 A_0\, x \left(1 - \frac{x}{2L}\right)$$

$$\frac{Kg}{\cancel{m^3}} \frac{1}{s^2} \cancel{m^2}\, \cancel{m} = \frac{N}{m} \checkmark$$

Continuum model

- differential: uniaxial deformations

$$(E A u')' + f(x) = 0$$

$$u(0) = 0 \quad \text{essential B.C.}$$

$$\underbrace{\underbrace{E u'(L)}_{\sigma(L)} A(L)}_{N(L)} = 0 \Rightarrow u'(L) = 0 \quad \text{natural B.C.}$$

- energy

$$U = \int_0^L \frac{1}{2} E\, u'^2 \, dx$$

$$V = -\int_0^L f(x)\, u(x)\, dx$$

$$\Pi = U + V$$

$\delta\Pi = 0 \Rightarrow$ equilibrium $\forall\, \delta u$ s.t. $\delta u = 0$ on $\partial\Omega_u$

$$\delta\Pi = \int_0^L \frac{1}{2} EA\, 2 u' \delta u' \, dx - \int_0^L f(x)\, \delta u \, dx$$

$$= \int_0^L (EA\, u' \delta u)' dx - \int_0^L (EA u')' \delta u \, dx - \delta V$$

$$0 = EA u' \delta u \Big|_0^L - \int_0^L (EA u')' \delta u \, dx - \int_0^L f(x) \delta u \, dx$$

$\Rightarrow (EAu')_{(L)} = 0$ natural B.C.

$(EAu')' + f(x) = 0 \quad 0 < x < L$

- PVD (alternatively)

$$\int_0^L A\sigma\, \delta\varepsilon = \int_0^L f(x)\, \delta u\, dx$$

- Approximate solution:
  - Ritz method: $\phi_i = x^i$

$$u \cong c_i \phi_i$$

$$\Pi = \int_0^L \frac{E}{2} A_0 \left(1 - \frac{x}{2L}\right)\left(\phi_i' c_i\right)^2 dx - \int_0^L \rho\omega^2 A_0 \left(1 - \frac{x}{2L}\right) \phi_i c_i\, dx$$

$$\frac{\partial \Pi}{\partial c_i} = 0$$

- One term: $u \sim c_1 x \quad u' = c_1$

$$\Pi = \int_0^L \frac{E A_0}{2}\left(1 - \frac{x}{2L}\right) c_1^2\, dx - \int_0^L \rho\omega^2 A_0 \left(1 - \frac{x}{2L}\right) c_1 x^2 d$$

$$\frac{\partial \Pi}{\partial c_1} = c_1 \int_0^L E A_0 \left(1 - \frac{x}{2L}\right) dx - \int_0^L \rho\omega^2 A_0 \left(1 - \frac{x}{2L}\right) x^2 dx$$

$$= c_1\, E \cancel{A_0}\left(L - \frac{L}{4}\right) - \rho\omega^2 \cancel{A_0}\left(\frac{L^3}{3} - \frac{L^3}{8}\right)$$

$$= C_1 \frac{3}{4} E - \frac{5\rho\omega^2}{24} L^2$$

$$\Rightarrow C_1 = \frac{\cancel{4}\,5}{3\,\cancel{24}\,6} \frac{\rho\omega^2 L^2}{E} = \frac{5}{18} \frac{\rho\omega^2 L^2}{E}$$

- Postprocessing:

$$u = C_1 \phi_1 = \frac{5}{18} \frac{\rho\omega^2 L^2}{E} x$$

$$\varepsilon = u' = \frac{5}{18} \frac{\rho\omega^2 L^2}{E}$$

$$\sigma = \frac{5}{18} \rho\omega^2 L^2$$

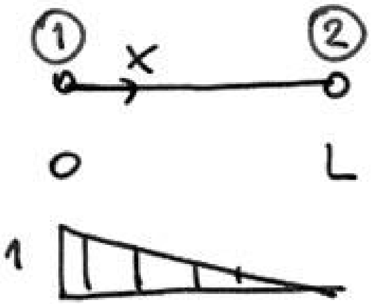
- $L = 5$, $\rho = 2700$, $\sigma_y = 500\ MPa$

$$\sigma = \frac{5}{18} \times 2700 \times 25 \times \omega^2 \leq 500\ 10^6$$

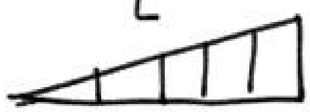
$$\omega^2 \leq 26666$$

$$\omega_{max} = 163 \frac{1}{sec}$$

• Finite elements



$$\phi_1 = 1 - \frac{x}{L}$$



$$\phi_2 = \frac{x}{L}$$

$$K = \int_0^1 E A_0 \left(1 - \frac{x}{2L}\right) \begin{bmatrix} (-1/L)^2 & (-1/L)(1/L) \\ (-1/L)(1/L) & (1/L)^2 \end{bmatrix} dx$$

$$= \frac{E A_0}{L^2}\left(L - \frac{L^2}{4}\right) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \frac{3}{4}\frac{E A_0}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$R = \int_0^L \rho \omega^2 A_0 \left(1 - \frac{x}{2L}\right) x \begin{Bmatrix} 1 - x/L \\ x/L \end{Bmatrix} dx$$

$$K_{11} \cancelto{0}{U_1} + K_{12} U_2 = R_1$$

$$K_{21} \cancelto{0}{U_1} + K_{22} U_2 = R_2$$

$$R_2 = \rho\omega^2 A_0\left(\frac{L^3}{3L} - \frac{L^4}{8L^2}\right) = \frac{5\rho\omega^2 A_0 L^2}{24}$$

$$\Rightarrow U_2 = \frac{5\rho\omega^2 \cancel{A_0} L^2}{24}\,\frac{4}{3}\,\frac{L}{E\cancel{A_0}} = \frac{5}{18}\frac{\rho\omega}{E}$$

$$u = \left(1-\frac{x}{L}\right)\times 0 + \frac{x}{\cancel{L}}\,\frac{5}{18}\,\frac{\rho\omega^2 L^{\cancel{3}\,2}}{E}$$

$$u = \frac{5}{18}\,\frac{\rho\omega^2 L^2 x}{E},$$ <u>same as Ritz</u>