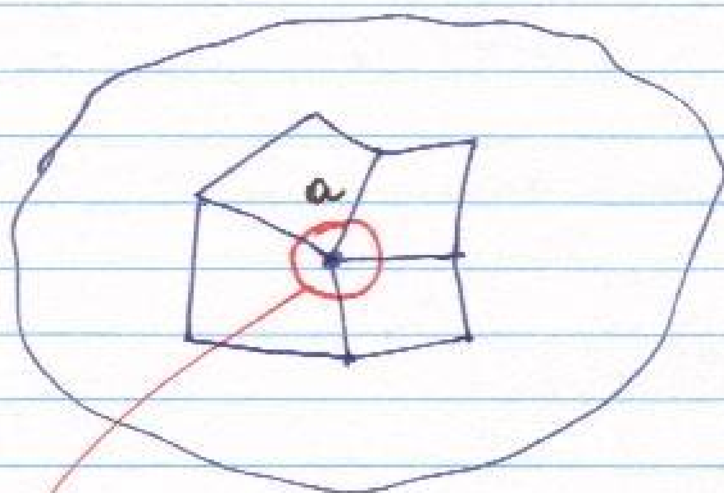
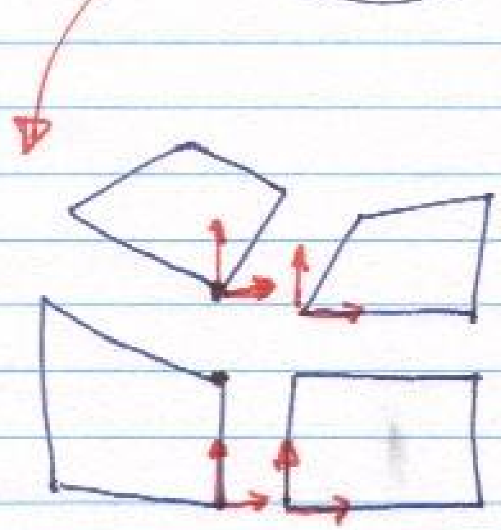
Properties of finite element solutions

• Nodal point equilibrium





$$\boxed{\Sigma \underline{f}^e = 0}$$

At a node, the sum of the element nodal forces is in equilibrium with the external loads.

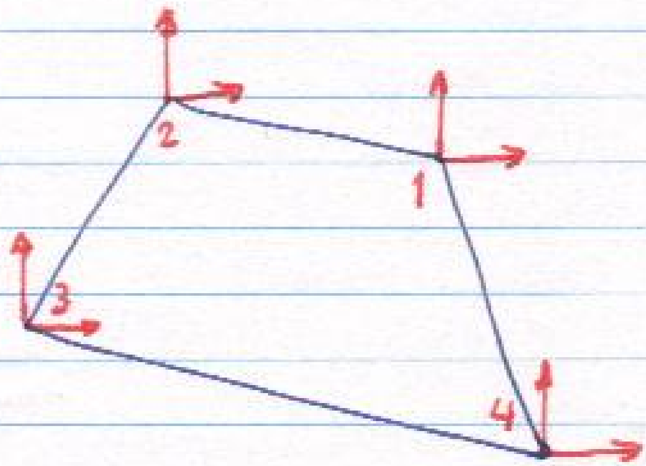
Follows directly from $\boxed{K U = R}$

$$\sum_e \int_{\Omega^e} B^T C B \{U^e\}\, dV = \sum_e \{R^e\}$$

$$\sum_e \int_{\Omega^e} B^T C \{\varepsilon\}\, dV = \sum_e \{R^e\}$$

$$\boxed{\sum_e \underbrace{\int_{\Omega^e} B^T \{\sigma\}\, dV}_{F^e} = \sum_e \{R^e\}}$$

- Element equilibrium



Each element is in equilibrium under its forces F^e

Related to convergence requirement on basis functions to allow for rigid body motions.

PVD for element "e" under a rigid-body virtual displacement. $\{\delta u\}$

$$\{F^e\} = \int_{\Omega^e} B^T \{\sigma\}\, dV$$

$$\{\delta u\}^T \{F^e\} = \{\delta u\}^T \int_{\Omega} B^T \{\sigma\}\, dV$$

$$= \int_{\Omega} \big(\underbrace{B\{\delta u\}}_{\delta\varepsilon = 0}\big)^T \{\sigma\}\, dV$$

$$= 0$$

Since $\{\delta u\}$ is a rigid-body but otherwise arbitrary virtual displacement field, this implies the forces are in equilibrium.

Interpretation of finite element analysis

- Continuum (structure) idealized as assembly of discrete elements connected at nodes.

- External forces are reduced to equivalent external nodal forces using PVD.

- The externally-applied nodal forces are equilibrated by the internal element nodal forces which are equivalent to the element internal stresses.

- Compatibility and constitutive relationship are satisfied exactly.

- Equilibrium is satisfied for the whole structure, at the nodes of the mesh and by every element of the mesh

- Equilibrium is not satisfied locally (at the differential level).