

Your name is: _____

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1 (30 pts.) Suppose the matrix A has reduced row echelon form R :

$$A = \begin{bmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \\ \text{(row 3)} \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) What can you say immediately about row 3 of A ?
- (b) What are the numbers a and b ?
- (c) Describe *all solutions* of $Rx = 0$. Circle the spaces that are the same for A as for R : (row space)(column space)(null space).

- 2 (30 pts.)** (a) Find the number c that makes this matrix singular (not invertible):

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 2 & 6 & c \end{bmatrix}$$

- (b) If $c = 20$ what are the column space $C(A)$ and the nullspace $N(A)$? Describe them in this specific case (not just repeat their definitions). Also describe $C(A^{-1})$ and $N(A^{-1})$ for the inverse matrix!
- (c) With $c = 20$ factor the matrix into $A = LU$ (lower triangular L and upper triangular U).

- 3 (40 pts.)** Suppose A is an m by n matrix of rank r .
- (a) If $Ax = b$ has a solution for every right side b , what is the column space of A ?
 - (b) In part (a), what are all equations or inequalities that must hold between the numbers m , n , and r .
 - (c) Give a specific example of a 3 by 2 matrix A of rank 1 with first row $[2 \ 5]$. Describe the column space $C(A)$ and the nullspace $N(A)$ completely.
 - (d) Suppose the right side b is the same as the first column in your example (part c). Find the complete solution to $Ax = b$.