

8.07 – Fórmulas básicas

$$\epsilon_{ijk}\epsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}, \quad \vec{a} \cdot \vec{b} = a_i b_i, \quad (\vec{a} \times \vec{b})_i = \epsilon_{ijk} a_j b_k$$

$$\det A = \epsilon_{i_1 i_2 \dots i_n} A_{1, i_1} A_{2, i_2} \dots A_{n, i_n}$$

$$\int_V \nabla \cdot \vec{F} = \int_S \vec{F} \cdot d\vec{a}, \quad \int_S \nabla \times \vec{F} = \int_\Gamma \vec{F} \cdot d\vec{l}$$

$$\nabla \times (\nabla \phi) = 0, \quad \nabla \cdot (\nabla \times \vec{A}) = 0$$

$$\delta(g(x)) = \sum_i \frac{\delta(x - x_i)}{|g'(x_i)|}, \quad g(x_i) = 0$$

$$\nabla r = \vec{e}_r \quad \nabla^2 \frac{1}{r} = -4\pi\delta(\vec{x})$$

$$\nabla \Phi = \frac{\partial \Phi}{\partial r} \vec{e}_r + \frac{1}{r} \frac{\partial \Phi}{\partial \theta} \vec{e}_\theta + \frac{1}{r \sin \theta} \frac{\partial \Phi}{\partial \phi} \vec{e}_\phi$$

$$\nabla^2 \Phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Phi}{\partial \phi^2}$$

$$|\vec{E}(r)| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \int_S \vec{E} \cdot d\vec{a} = \frac{Q_{encl}}{\epsilon_0}, \quad \vec{E} = -\nabla V \quad \nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$W = \frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|\vec{x}_i - \vec{x}_j|} \rightarrow \frac{1}{8\pi\epsilon_0} \int d^3 x d^3 x' \frac{\rho(\vec{x}) \rho(\vec{x}')}{|\vec{x}_i - \vec{x}_j|} \rightarrow w = \frac{\epsilon_0}{2} |\vec{E}|^2$$

$$Q = CV \rightarrow Q_i = C_{ij} V_j$$

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int d^3 x' G(\vec{x}, \vec{x}') \rho(x') + \frac{1}{4\pi} \oint_S \left[G(\vec{x}, \vec{x}') \frac{\partial V}{\partial n'} - V(\vec{x}') \frac{\partial G(\vec{x}, \vec{x}')}{\partial n'} \right] da'$$

$$\nabla_{\vec{x}'}^2 G(\vec{x}, \vec{x}') = -4\pi\delta(\vec{x} - \vec{x}')$$

$G(\vec{x}, \vec{x}')$ = “potencial” en \vec{x}' debido a una unidad de carga en \vec{x} , cuando ...

Imagen fijada para esfera: $(q, y), (q', y')$ con $y y' = a^2, \quad q' = -\frac{a}{y} q$

$$\left\{ \begin{array}{l} \cos \alpha x \\ \sin \alpha x \end{array} \right\} \left\{ \begin{array}{l} \cos \beta y \\ \sin \beta y \end{array} \right\} \left\{ \begin{array}{l} \cosh \gamma z \\ \sinh \gamma z \end{array} \right\} \quad \gamma^2 = \alpha^2 + \beta^2 \quad \text{una solución cartesiana de Laplace}$$

$$\{r^l, r^{-(l+1)}\} P_l(\cos \theta) \quad \text{soluciones acimutales de Laplace}$$

$$\frac{1}{|\vec{x} - \vec{x}'|} = \sum_{l=0}^{\infty} \frac{r_{<}^l}{r_{>}^{l+1}} P_l(\cos \gamma), \quad \frac{1}{\sqrt{1 - 2hx + h^2}} = \sum_{l=0}^{\infty} h^l P_l(x)$$

$$P_l(1) = 1 \quad P_l(-x) = (-1)^l P_l(x) \quad \int_{-1}^1 dx P_{l'}(x) P_l(x) = \frac{2}{2l+1} \delta_{ll'}$$

$$S'_{i_1 \dots i_n} = (\det R) R_{i_1 j_1} \dots R_{i_n j_n} S_{j_1 \dots j_n} \quad (\text{pseudo) tensor bajo rotaci3n}$$

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{|\vec{x}|} + \frac{\vec{p} \cdot \vec{x}}{|\vec{x}|^3} + \frac{1}{2} \frac{x_i x_j}{|\vec{x}|^5} Q_{ij} + \dots \right]$$

$$Q = \int d^3x \rho(\vec{x}), \quad p_i = \int d^3x \rho(\vec{x}) x_i \quad Q_{ij} = \int d^3x \rho(\vec{x}) (3x_i x_j - \delta_{ij} |\vec{x}|^2),$$

$$\vec{E}_{dip} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} (3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p}) \quad \rightarrow \quad \vec{E}_{dip} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}), \quad \vec{p} = p\hat{z}$$

$$\vec{J} d^3x \Leftrightarrow \vec{K} da \Leftrightarrow I d\vec{l}, \quad \vec{J} = \rho \vec{v}, \quad \nabla \cdot \vec{J} = 0, \quad \frac{d\vec{p}}{dt} = q \vec{v} \times \vec{B}$$

$$d\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3}, \quad \vec{B} = \frac{\mu_0 I}{4\pi} \nabla \Omega$$

$$\vec{B} = \nabla \times \vec{A}, \quad \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{\vec{J}(\vec{x}')}{|\vec{x} - \vec{x}'|}, \quad \nabla \cdot \vec{A} = 0,$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}, \quad \int_{\Gamma} \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enlazado}}, \quad \nabla \cdot \vec{B} = 0$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{x}}{|\vec{x}|^3}, \quad \vec{B} = -\frac{\mu_0}{4\pi} \nabla \left(\frac{\vec{m} \cdot \vec{x}}{|\vec{x}|^3} \right), \quad \vec{m} = \frac{1}{2} \int d^3x \vec{x} \times \vec{J}(\vec{x}), \quad \vec{m} = I \vec{A}$$

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi} \frac{1}{r^3} (3(\vec{m} \cdot \hat{r})\hat{r} - \vec{m}) \quad \rightarrow \quad \vec{B}_{dip} = \frac{\mu_0}{4\pi} \frac{m}{r^3} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta}), \quad \vec{m} = m\hat{z}$$

$$\mathcal{E} = \int_{\Gamma} (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a}$$

$$W = \frac{1}{2\mu_0} \int \vec{B} \cdot \vec{B} d^3x = \frac{1}{2} \int \vec{A} \cdot \vec{J} d^3x$$

$$M_{ij} = \frac{\mu_0}{4\pi} \oint_{C_i} \oint_{C_j} \frac{d\vec{l}_i \cdot d\vec{l}_j}{|\vec{x}_i - \vec{x}_j|} = \frac{1}{I_j} \int_{S_i} \vec{B}_{ij}(\vec{x}) \cdot d\vec{a}, \quad M_{ij} = M_{ji}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\frac{dE_{mec}}{dt} = \int_V d^3x \vec{J} \cdot \vec{E}, \quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\frac{dE_{mec}}{dt} + \frac{d}{dt} \int_V d^3x \frac{1}{2} (\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) = - \int_S \vec{S} \cdot d\vec{a}$$

$$\frac{dP_{mec}}{dt} = \int_V d^3x (\rho \vec{E} + \vec{J} \times \vec{B})$$

$$T_{\alpha\beta} = \epsilon_0 \left[E_\alpha E_\beta + c^2 B_\alpha B_\beta - \frac{1}{2} (\vec{E} \cdot \vec{E} + c^2 \vec{B} \cdot \vec{B}) \delta_{\alpha\beta} \right]$$

$$\left(\frac{dP_{mec}}{dt} + \frac{d}{dt} \int_V d^3x \epsilon_0 (\vec{E} \times \vec{B}) \right)_i = + \int_S T_{ij} n_j da$$

$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}$$

$$\Phi(\vec{x}, t) = \frac{1}{4\pi\epsilon_0} \int d^3x' \frac{\rho(\vec{x}', t - \frac{|\vec{x}-\vec{x}'|}{c})}{|\vec{x}-\vec{x}'|}, \quad \vec{A}(\vec{x}, t) = \frac{\mu_0}{4\pi} \int d^3x' \frac{\vec{J}(\vec{x}', t - \frac{|\vec{x}-\vec{x}'|}{c})}{|\vec{x}-\vec{x}'|}$$

$$\vec{E}(\vec{x}, t) = \vec{\mathcal{E}} e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

$$\vec{B}(\vec{x}, t) = \vec{\mathcal{B}} e^{i(\vec{k} \cdot \vec{x} - \omega t)}$$

$$\vec{n} \cdot \vec{\mathcal{E}} = \vec{n} \cdot \vec{\mathcal{B}} = 0, \quad \vec{\mathcal{B}} = \sqrt{\mu\epsilon} \vec{n} \times \vec{\mathcal{E}} \quad \vec{\mathcal{E}} = Z \vec{\mathcal{H}} \times \vec{n}, \quad Z = \sqrt{\mu/\epsilon}$$

$$\langle \vec{E} \circ \vec{B} \rangle = \frac{1}{2} \text{Re}(\vec{\mathcal{E}} \circ \vec{\mathcal{B}}^*)$$

$$\text{Radiación : } e^{-i\omega t} \quad \vec{H}(\vec{x}) = \frac{1}{\mu_0} \nabla \times \vec{A}(\vec{x}) \quad \vec{E}(\vec{x}) = \frac{iZ_0}{k} \nabla \times \vec{H}(\vec{x})$$

$$\vec{A} = -\frac{i\mu_0}{4\pi} \omega \vec{p} \frac{e^{ikr}}{r}, \quad \frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 |(\vec{n} \times \vec{p}) \times \vec{n}|^2, \quad P = \frac{c^2 Z_0}{12\pi} k^4 |\vec{p}|^2$$

$$\text{radiación dipolar magnética} \quad \frac{dP}{d\Omega} = \frac{dP}{d\Omega}(\vec{p} \rightarrow \vec{m}/c), \quad P = P(\vec{p} \rightarrow \vec{m}/c)$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}, \quad \nabla \cdot \vec{D} = \rho \quad \text{material lineal : } \vec{D} = \epsilon \vec{E}$$

$$\rho_{pol} = -\nabla \cdot \vec{P}, \quad \sigma_{pol} = \vec{n} \cdot \vec{P}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}, \quad \nabla \times \vec{H} = \vec{J}, \quad \text{material lineal : } \vec{B} = \mu \vec{H}$$

$$\vec{J}_b = \nabla \times \vec{M}, \quad \vec{K}_b = \vec{M} \times \vec{n}$$

$$\vec{H} = -\nabla\Phi_M \rightarrow \nabla^2\Phi_M = -\rho_M, \quad \rho_M = -\nabla \cdot \vec{M}, \quad \sigma_M = \vec{M} \cdot \vec{n}$$